# Regression

Statistic Modeling & Causal Inference – Oswald | Ramirez-Ruiz

# Agenda



- Regression from a causal perspective
- Back-door criterion

- Regression in R
- DAGs in R

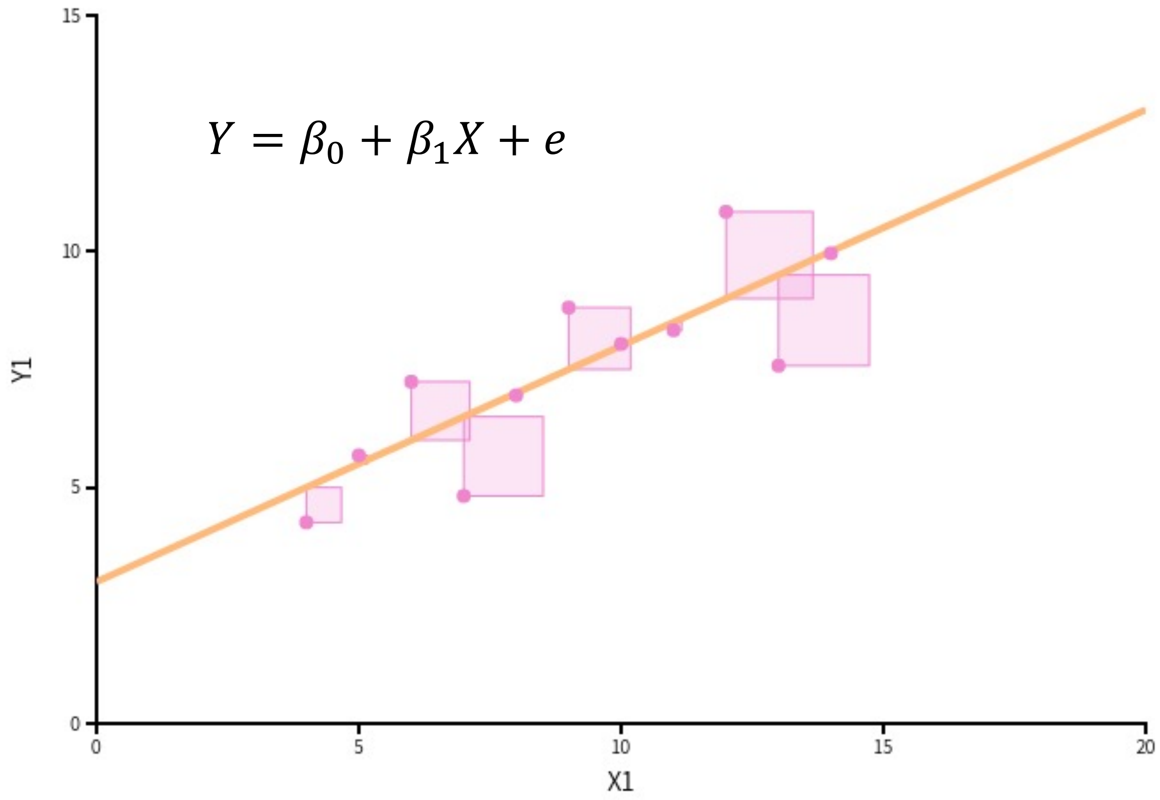
# Ordinary Least Squares (OLS)



- Minimizing sum of squares of residuals (differences between observed and predicted values)
- Finding the best (linear) guess for y given a particular x value

Minimize $\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$

$$Y = \beta_0 + \beta_1 X + e$$




https://seeing-theory.brown.edu/regression-analysis/index.html

# Regression Coefficients



## Calculation of intercept

Y value given X = 0

$$\hat{\beta}_0 = \bar{y} - \beta_1 \bar{x}$$

## Calculation of slope

How much does Y change when X increases by 1 unit

$$\hat{\beta}_1 = \frac{cov(x, y)}{var(x)} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

(in bivariate regression)

$$Y = \beta_0 + \beta_1 X + e$$

# Omitted Variable Bias (OVB)



- Bivariate relationships can be confounded by other variables
- Occurs when *Z* is a common cause of both *X* and *Y*

→Include Z in regression to (partially) deal with the issue

Multiple Regression

$$Y = \beta_0 + \beta_1 X + ??? + e$$

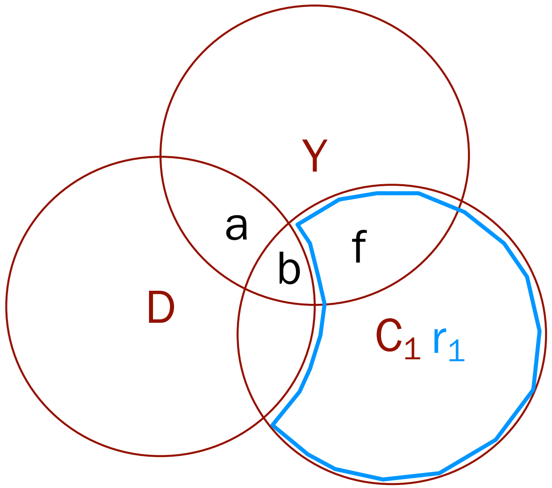
# Multiple Regression



- Multiple regression only uses the unique variation in each regressor ($X_i$) not explained by other regressors
- Thus, $\beta_2$ can be estimated by $\hat{\beta}_2 = \frac{\sum \hat{r}_{i1} y_i}{\sum \hat{r}_{i1}^2}$

$$Y = \beta_0 + \beta_1 D + \beta_2 C_1 + e$$

$$Y = \beta_0 + \beta_1 D + \beta_2 C_1 + \beta_3 C_2 + \ldots + e$$




- r is the residual from a regression of $C_1$ on the other explanatory variables (in this case only D)
- $r_{i1}$ is $c_{i1}$ after the effects of $d_i$ *(and potentially all other* $c_{ij}$*)* have been "partialled out"

# Regression from a POF perspective



- Regression can be utilized without thinking about causes as a predictive or summarizing tool.
- It would not be appropriate to give causal interpretations to any, unless we establish the fulfilment of certain assumptions.

$$Y_i = \beta_0 + \beta_1 D + e_i \quad ?$$

$$E(Y^0 | D = 0) = \beta_0$$

$$E(Y^1 | D = 1) = \beta_0 + \beta_1$$

$$\beta_1 = E(Y^1 | D = 1) - E(Y^0 | D = 0)$$

$$= NATE$$

# Regression Error Terms



$$Y_i = \beta_0 + \beta_1 D + e_i \quad \neq \quad Y_i = \beta_0 + \beta_1 X + r_i$$

- Error term in causal perspective: "summary" random variable representing all causes other than D (and other modeled regressors)

- regression residual r, which is uncorrelated with the regressors by construction

→ Only if *D* and *e* were independent (e.g., due to random assignment of *D*), the regression estimate of $\beta_1$ could be given a causal interpretation /$\beta_1$ = *ATE*

# Back to OVB



- Violations of the assumption that *D* and *e* are independent:

→ OVB problem

"true" causal model: $Y_i = \beta_0 + \beta_1 D_i + \beta_2 Z_i + e_i,$

Fitted empirical model: $Y_i = \gamma_0 + \gamma_1 D_i + r_i$

Relationship between Z and D: $D_i = \delta_0 + \delta_1 Z_i + u_i$

OVB: $\gamma_1 - \beta_1 = \beta_2 \delta_1$

→ does Z influence Y? $\beta_2$

→ does Z influence D? $\delta_1$

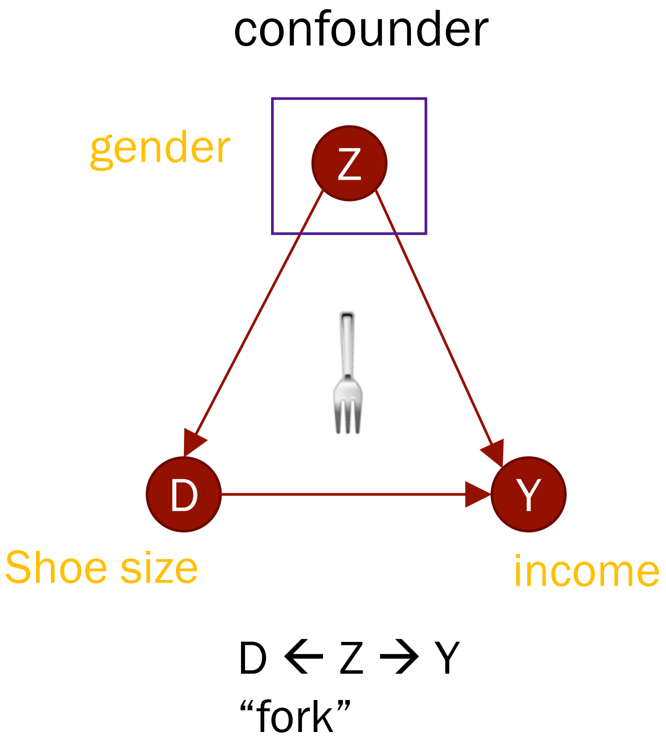
# Tackling OVB with Regression



- "Control for" / "Condition on" confounders by including them in the model

- find the factors responsible for different baseline values (or differential treatment effects), and to include these variables in the equation in the hope that an unbiased estimate of $\beta_1$ is obtained

- But: we need to include all relevant covariates and there has to be a large enough overlap in covariate values across different values of D ("common support")



# Selecting Covariates



- Draw DAG
- Write down all paths between D and Y
- Identify conditions that satisfy back-door-criterion
- Control for the identified variables in model
- Only interpret D causally! The status of covariates is path-specific


Photo by Kaitlan Balsam on Unsplash

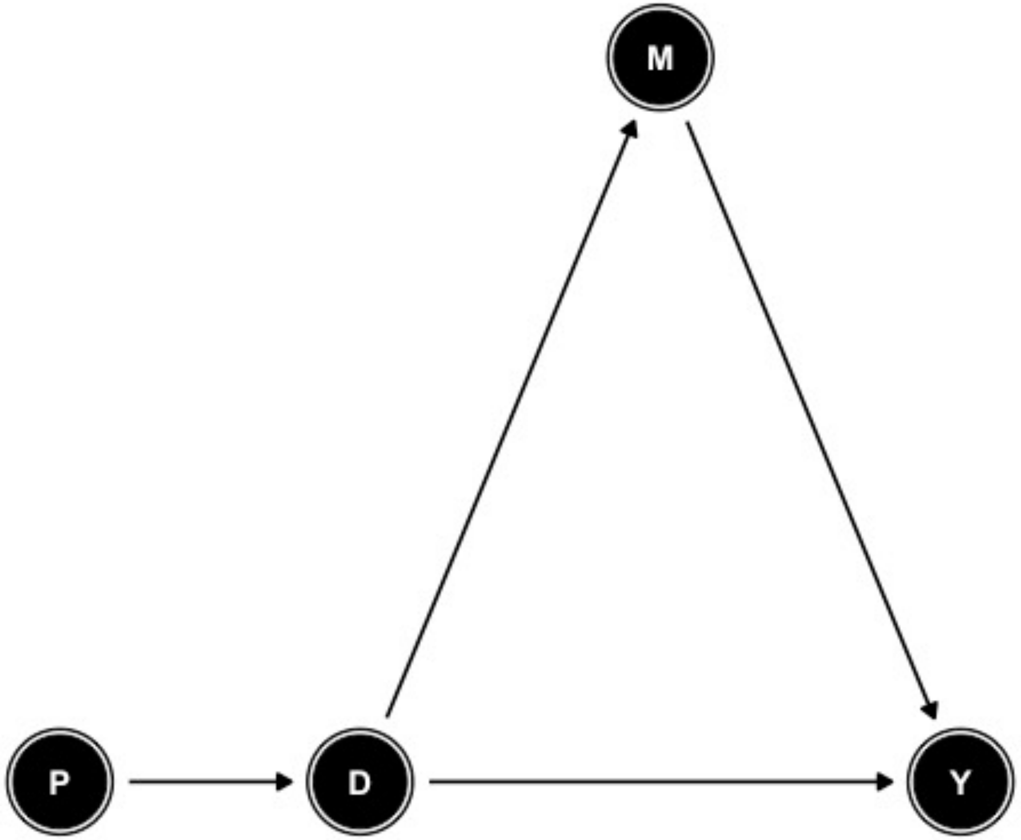
# Back-door criterion



- Potential shortcut to formal adjustment criterion
- Focus on non-causal paths that start with an arrow into D (back-door-paths)
- To identify the total effect of $D$ on $Y$, you need to condition on observed variables $Z$ so that
  - no element of $Z$ is a descendant of $D$, and
  - $Z$ blocks all back-door paths from $D$ to $Y$
- Remember: Confounders, Mediators, Colliders…

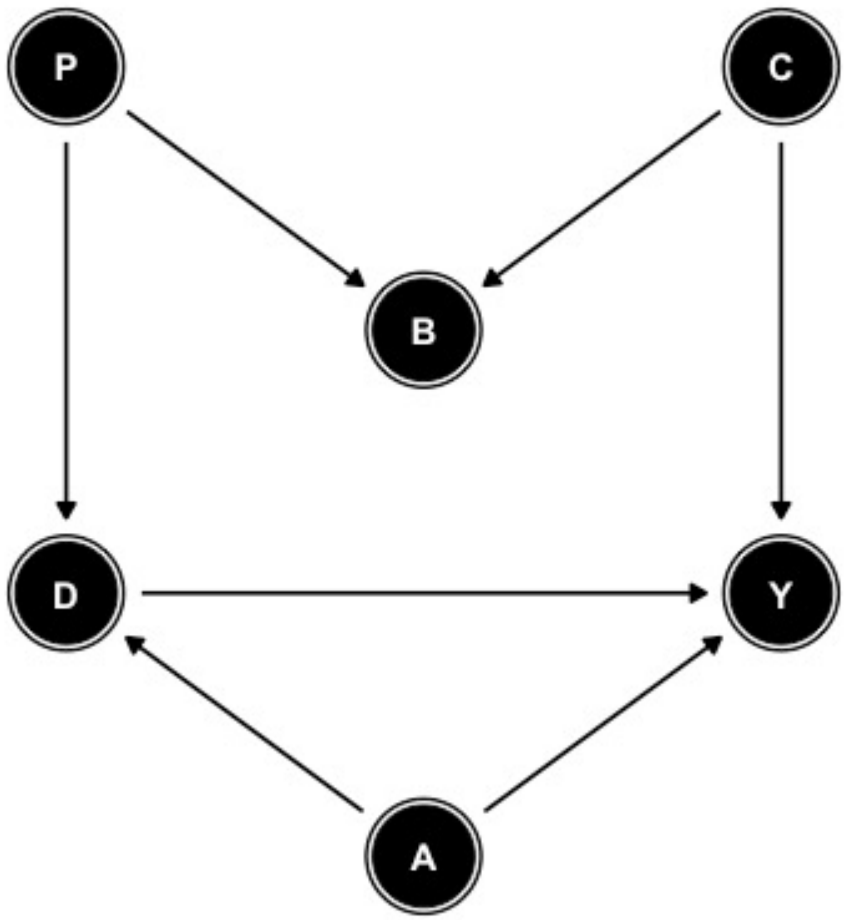
# Example 1





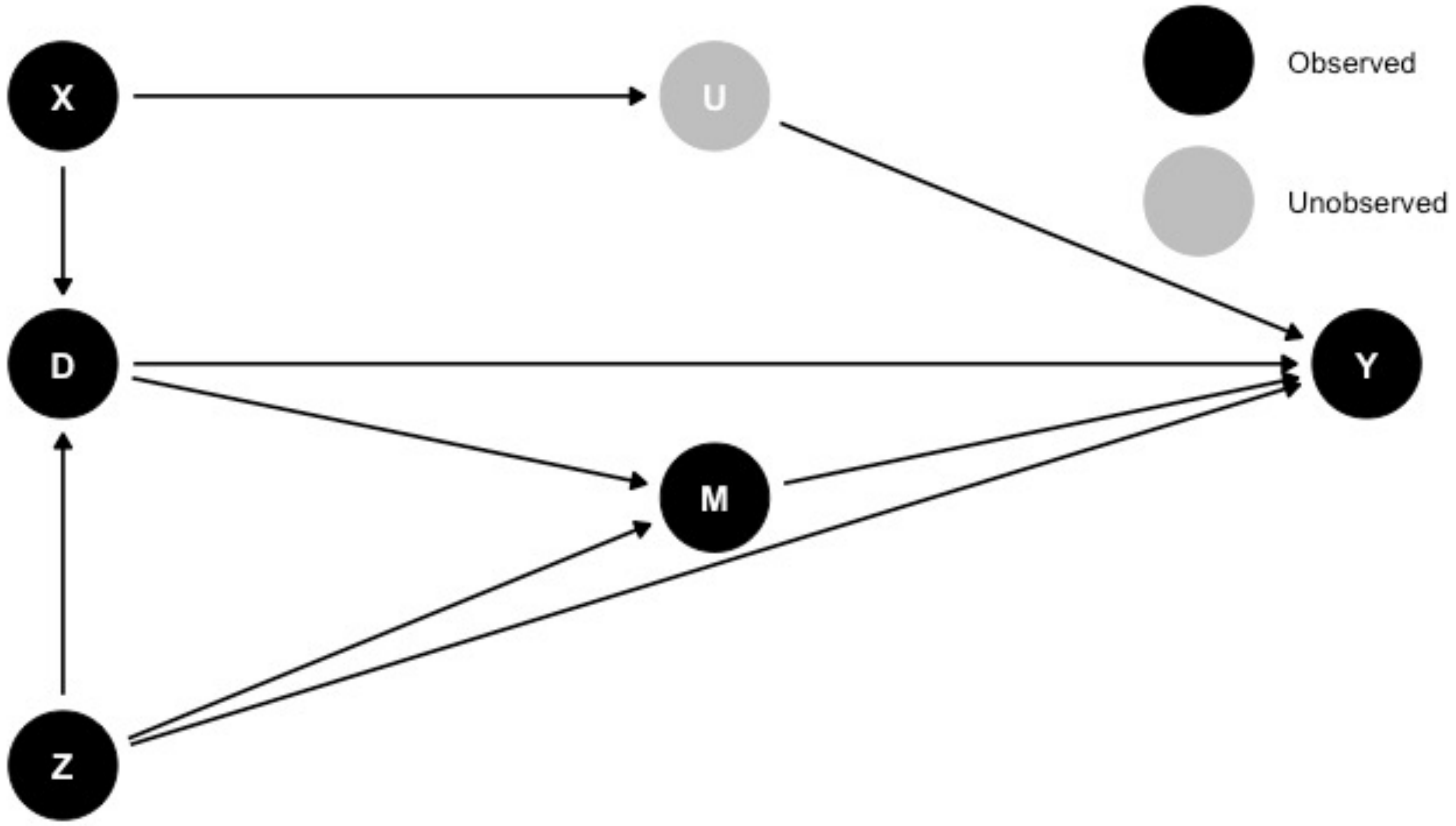
# Example 2





# Example 3





# Further Ressources



For any coding issues – Stackoverflow

Hertie’s Data Science Lab – Research Consulting