Difference in Difference

Statistic Modeling & Causal Inference | Oswald & Ramirez-Ruiz

Agenda

- Lecture review
  - Unit and Time comparisons
  - Difference in Difference
  - Parallel Trends Assumption
  - Estimation & Interpretation

- Diff-in-diff in R

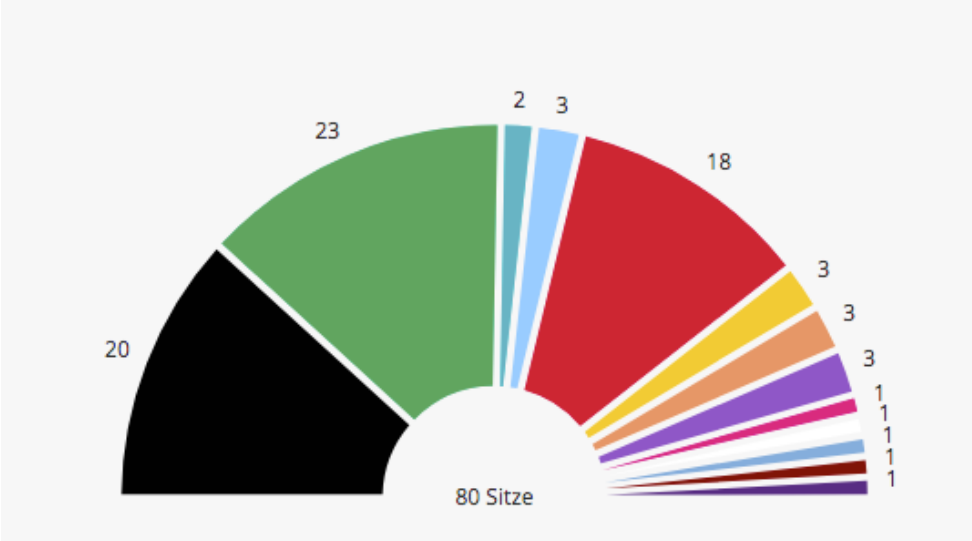
Motivation

- Until now: focus on treatment / control comparison without consideration of **time**
- Time important for causality: **cause always precedes the effect**
- By considering both, units and time, we can:
  - Compare individuals to themselves, to **account for units' characteristics** that affect both outcome and treatment (~ permanent differences between groups).
  - Compare how outcomes for different units change across time, to **account for characteristics of different periods** (~ trends in Y that affect all units, regardless of treatment).

Example case

Effect of COVID-19 on electoral outcomes

- Focus: municipal elections in Bavaria in March 2020 (Leininger & Schaub, 2020)
- Treatment: cases per county
- Outcome: CSU vote shares
- Comparison over time: 2014 and 2020
- Comparison between units: two different Counties



https://www.wahlen-muenchen.de/ergebnisse/20200315stadtratswahl/index.html#w_8117_18028

Unit & Time Comparisons

Assume we have a data set with **two outcome measurements:** before and after treatment.
As usual, we have a problem of not knowing **counterfactuals.** We could:

Compare the **treatment and control groups**

	CSU vote shares		
Unit	Y_{2014}	Y_{2020}	D
County A	42.1	38.5	0
County B	41.2	40.2	1
...	...	...	...

This assumes the PO of control group is the same as the counterfactual PO for those being treated.

or

Compare **before and after treatment for the treatment group**

	CSU vote shares		
Unit	Y_{2014}	Y_{2020}	D
County A	42.1	38.5	0
County B	41.2	40.2	1
...	...	...	...

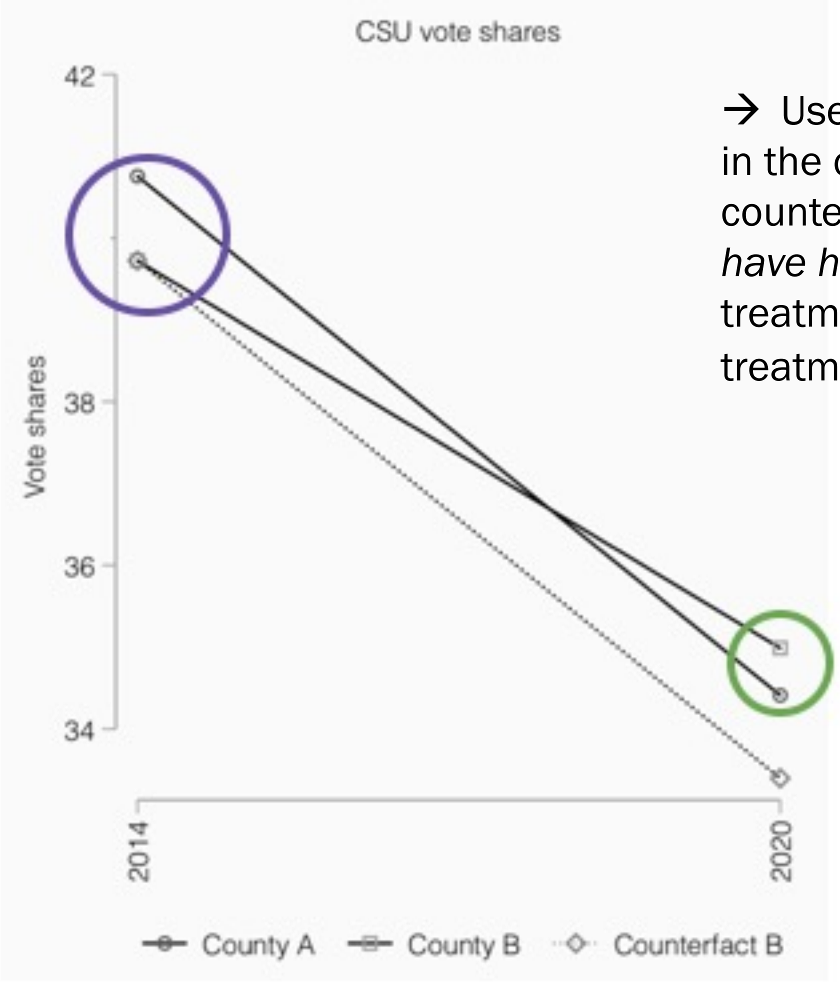
This assumes no change in average PO over time.

Difference-in-Difference

Or consider both!

1. Get the difference between the treatment and control group **after** treatment
2. Get the difference between the treatment and control group **before** treatment
3. Subtract the second difference from the first

CSU vote shares				
Unit	Y_{2014}	Y_{2020}	D	$\Delta Y_{2020-2014}$
County A	42.1	38.5	0	-3.6
County B	41.2	40.2	1	-1
	-0.9	1.7		2.6

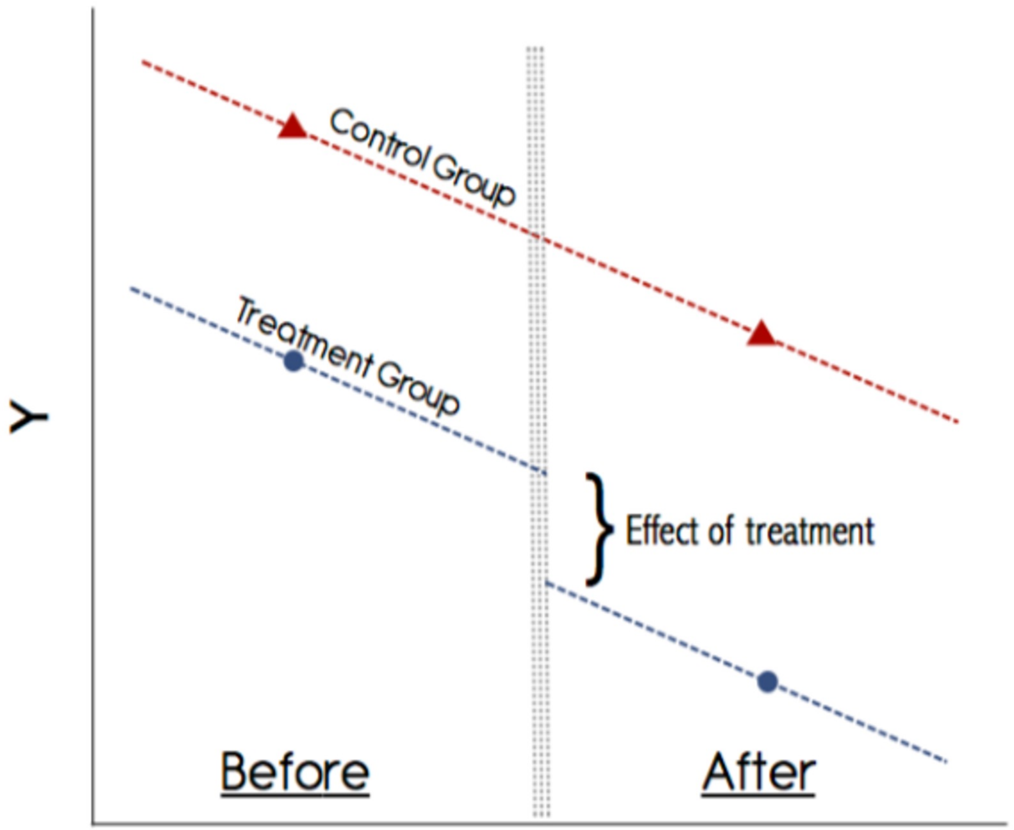


→ Use overtime difference in the control group as a counterfactual of *what would have happened* in the treatment group, had the treatment no taken place

Difference-in-Difference

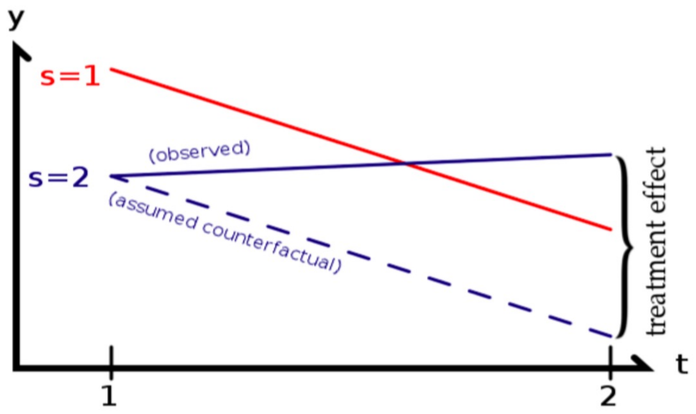
- Main idea:
  - Sometimes treatment and control units move in parallel in the absence of treatment.
  - When they do, we can see how much do the treated units diverge from the post-treatment expected path, compared to the control units.
  - We can estimate the treatment effect as the divergence from the expected outcome of the treatment group in the absence of treatment.

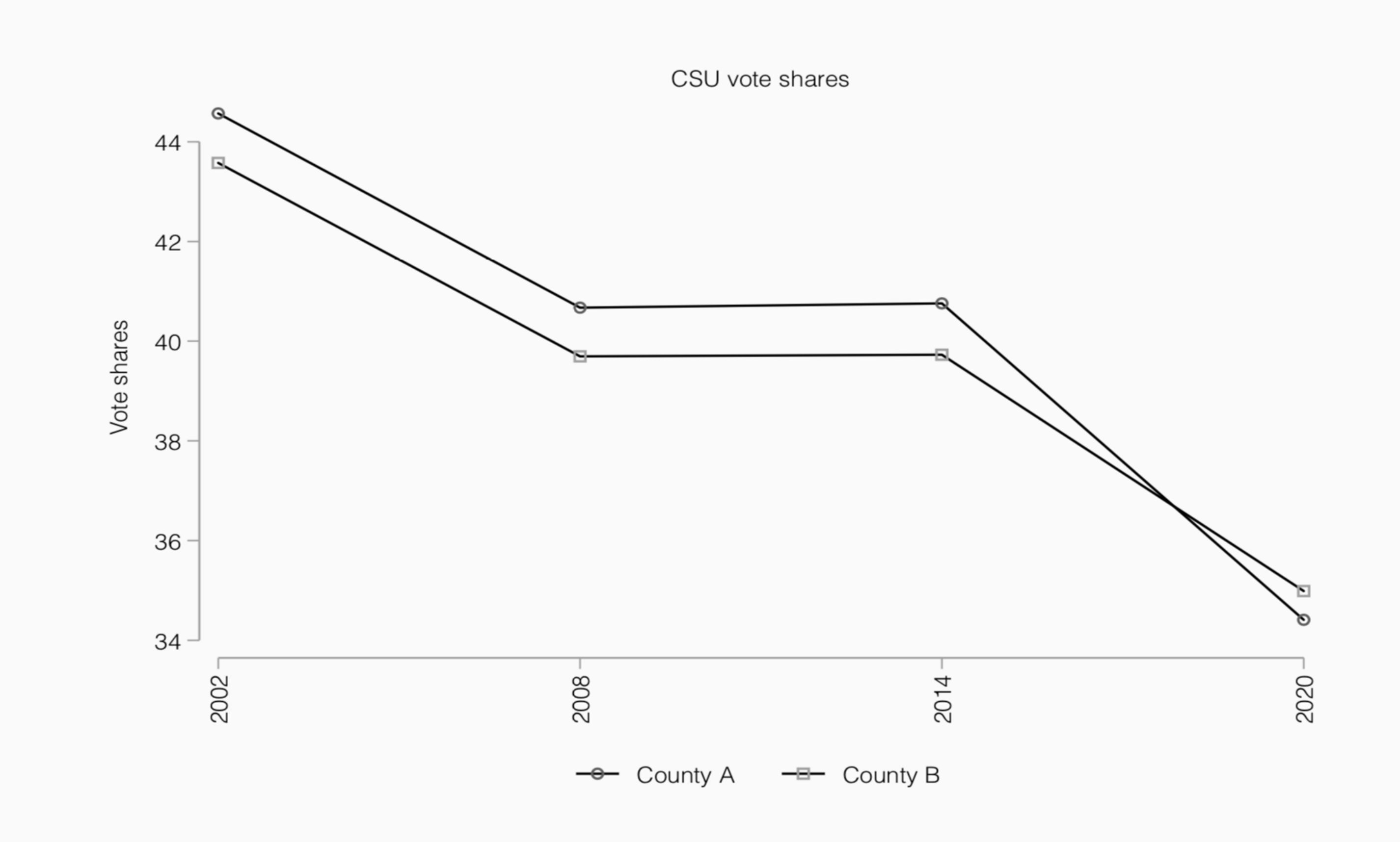
$$Y_{it} = \beta_0 + \underbrace{\beta_1 D_i^*}_{\text{Unit Main effect}} + \underbrace{\beta_2 P_t}_{\text{Time Main effect}} + \overbrace{\beta_{DD} D_i^* \times P_t}^{\text{Interaction Effect Unit x Time}} +$$



Parallel Trends Assumption

- Use the overtime difference in the control group as a counterfactual
- Assume that observed overtime changes in the control group reflect, on average, unobserved changes in the treatment group in the absence of treatment.





Estimating DiD

1. **Manually**, using average outcome values for subgroups defined by D and t.

$$DiD = \{E[Y_{1c}|D=1,t=1] - E[Y_{0c}|D=0,t=1]\} - \{E[Y_{1c}|D=1,t=0] - E[Y_{0c}|D=0,t=0]\}$$

2. Calculate **first differences** and regress on D. → wide format data

$$\Delta Y_{ct_0-t_1} = \alpha + \delta D_c + \Delta_{v_c}$$

3. Regression formulation of the **DiD model**. → long format data

$$Y_{it} = \beta_0 + \beta_1 D_i^* + \beta_2 P_t + \beta_{DD} D_i^* \times P_t + q_{it}$$

Interpreting Results

Regression output:

	Share CSU
Treat	-1.03
	(1.56)
Post	-6.34***
	(0.72)
Treat × Post	1.61**
	(0.79)
Intercept	40.76***
	(1.39)
N	192
R^2	0.16

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

D^*	$t = 0$	$t = 1$	Difference
1	$\beta_0 + \beta_1$	$\beta_0 + \beta_1 + \beta_2 + \beta_{DD}$	$\beta_2 + \beta_{DD}$
0	β_0	$\beta_0 + \beta_2$	β_2

Interaction Effect
Unit x Time

$$Y_{it} = \beta_0 + \beta_1 D_i^* + \beta_2 P_t + \beta_{DD} D_i^* \times P_t + q_{it}$$

Unit
Main effect

Time
Main effect

Data Formats

Wide

Wide format table			
Unit c	Y_{c2014}	Y_{c2020}	D_c
County A	42.1	38.5	0
County B	41.2	40.2	1
...	...	...	...

- Only one row per individual or unit.
- Outcome values included in different variables, by year.

Long

Long format table			
Unit c	Year t	Y_c	D_c
County A	2014	42.1	0
County A	2020	38.5	0
County B	2014	41.2	1
County B	2020	40.2	1
...	...	...	...

- One column for every variable.
- One row for every unique observation

Parallel Trends Violations

- **Selection** and **Targeting**
  - Units may self-sort for reasons that are not random
  - Policies may be targeted at units in a non-random way
- **Compositional differences** across time
  - The composition of a sample might change in ways that confound the treatment effect.
- **Long-term effects** vs. reliability
  - Parallel trends is more likely to hold in the short term.
- **Functional form** dependence
  - DD is more reliable if the treatment and control groups are more similar at baseline.

Parallel Trends Diagnostics

1. **Pre-treatment trends** in the outcome
2. Placebo test using **previous periods**
3. Placebo test using **alternative outcomes**
4. **Placebo outcomes**

Further Resources

For any coding issues – Stackoverflow

Hertie's Data Science Lab – Research Consulting